COMP 4680/8650: Advanced Topics in Statistical Machine Learning

Written Assignment I

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1. Given neighbor has 2 children i.e.

. *(B = Boy, G= Girl)*

So we have 2 possibilities i.e. .

Therefore,

So we have 3 possibilities i.e. .

Therefore,

1. Now, we are told that neighbor has boy or boys among their kids, which means at least one of the children is a boy.

So we have 3 possibilities where we have i.e. .

But we have 2 events where i.e. .

Therefore,

Comparing with case a), our belief of knowing that exactly one child is a girl increases as we have observed that one of the children is a boy, so we can cancel out the case where both children are girls.

Comparing with case b), our belief of knowing that exactly one child is a girl decreases as we are ignoring the case where both children are girls.

1. Now, we saw one of the children running and it turned out to be a boy.

Therefore we saw one of the following:

* First boy of BB
* Second boy of BB
* Boy of BG
* Boy of GB

Out of the above 4 possible events, only 2 events i.e. .

Therefore,

1. From Bayes’ theorem, we know that

where

1. Here,

We are given,

and Prosecutor is saying,

i.e.

This is only possible if .

This is the mistake with prosecutor’s argument as the possibility of is very rare.

1. Here we are given,

Again from Bayes’ theorem,

For this to be possible should be at least 100, which is just not possible.

The mistake with defender’s argument is that he did not consider the prior while making his statement.

1. Let,

G represent the fraction of the cabs of the Green company.

B represent the fraction of the cabs of the Blue company.

SG represent the event of *seeing a cab* from Green company.

SB represent the event of *seeing a cab* from Blue company.

Given,

To find,

Using Bayes’ theorem,

Therefore, we can say probability of the cab involved in accident was a Blue cab as claimed by the eyewitness is 94.4% which is almost true.

1. The functions are:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A | B | f1(A,B)  = XOR(A,B) | B | C | f2(B,C)  = XOR(B,C) | C | A | f3(C,A)  = XOR(C,A) |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |

From the above table, we see and for all and .

0

0

0

0

0

0

0

1

1

1

1

1

1

0

**A**

**C**

**B**

**A**

1

0

1

0

1

1

From the above diagram,

The paths that can lead us to  *(4th stage)* from *(in 1st stage)* are:

* **(A=0) -> (B=0) -> (C=0) -> (A=0)**
* **(A=0) -> (B=0) -> (C=1) -> (A=0)**
* **(A=0) -> (B=1) -> (C=0) -> (A=0)**
* **(A=0) -> (B=1) -> (C=1) -> (A=0)**

Similarly the paths that can lead us to *(4th stage)* from *(1st stage)* are:

* **(A=1) -> (B=0) -> (C=0) -> (A=1)**
* **(A=1) -> (B=0) -> (C=1) -> (A=1)**
* **(A=1) -> (B=1) -> (C=0) -> (A=1)**
* **(A=1) -> (B=1) -> (C=1) -> (A=1)**

From above, we see and this can be proved for other realizations also.

1. From Bayes’ theorem,
2. i) Here we are given . But we are not told anything about conditional independence of and i.e. , so we can’t write . Therefore instead of , we need . So we don’t have sufficient information.

ii) Here we are given all the necessary terms to calculate i.e. . So, this set of numbers is sufficient.

iii) As in i), we are not sure if will result to . Therefore knowing will not help.

1. As we are given, , therefore we can write

Out of i), ii) and iii), all are sufficient as we are given as required in the numerator. We also know , therefore we can write . Now marginalizing over we can obtain .

Therefore we can write:

1. For conditional independence in graphs, we can use d separation.

We know if a probability distribution factorizes according to a directed acyclic graph and if A, B and C are disjoint subsets of nodes such that A is d-separated from B by C, then the distribution satisfies .

This statement is TRUE and can be proved as follows:

Y

Z

U

W

X

Here as are both observed tail-tail nodes and is an unobserved head-head node. Therefore path from to is blocked and hence and are d-separated by or or .

Now,

Y

Z

U

W

X

Here as is an observed tail-tail node and is an unobserved head-head node. Therefore path from to is blocked and hence and are d-separated by or .

Now considering both scenarios i.e. :

Y

Z

U

W

X

We can say as is an observed tail-tail node and hence information from to or cannot reach. Hence, and are d-separated by .

This statement is FALSE and can be proved as follows:

W

Z

Y

U

X

In the above graph as is an unobserved head-head node and therefore path from to is blocked. Hence and are d-separated by .

Now,

X

W

Z

Y

U

In the above graph as is an unobserved head-head node and therefore path from to is blocked. Hence and are d-separated by .

Now, considering both scenarios i.e. ,

X

W

Z

Y

U

We see that path from to is no more blocked as are both observed head-head nodes and is an unobserved tail-tail node. Hence and are not d-separated.

Therefore,

**7.**

c

f

e

d

b

a

g

**Elimination order:**

1. Eliminating

a

b

c

f

d

g

g

1. Eliminating

c

b

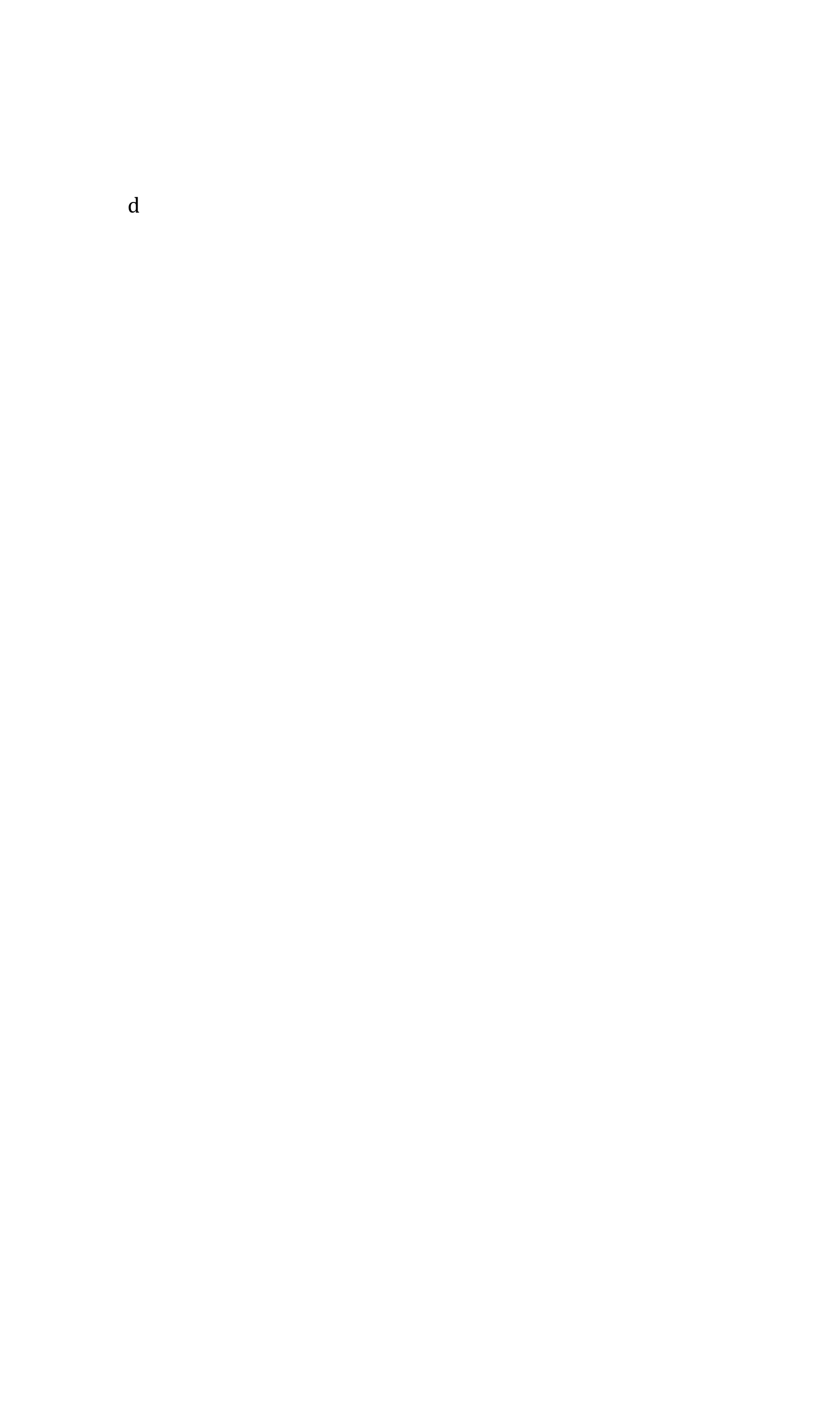
f

d

g

1. Eliminating

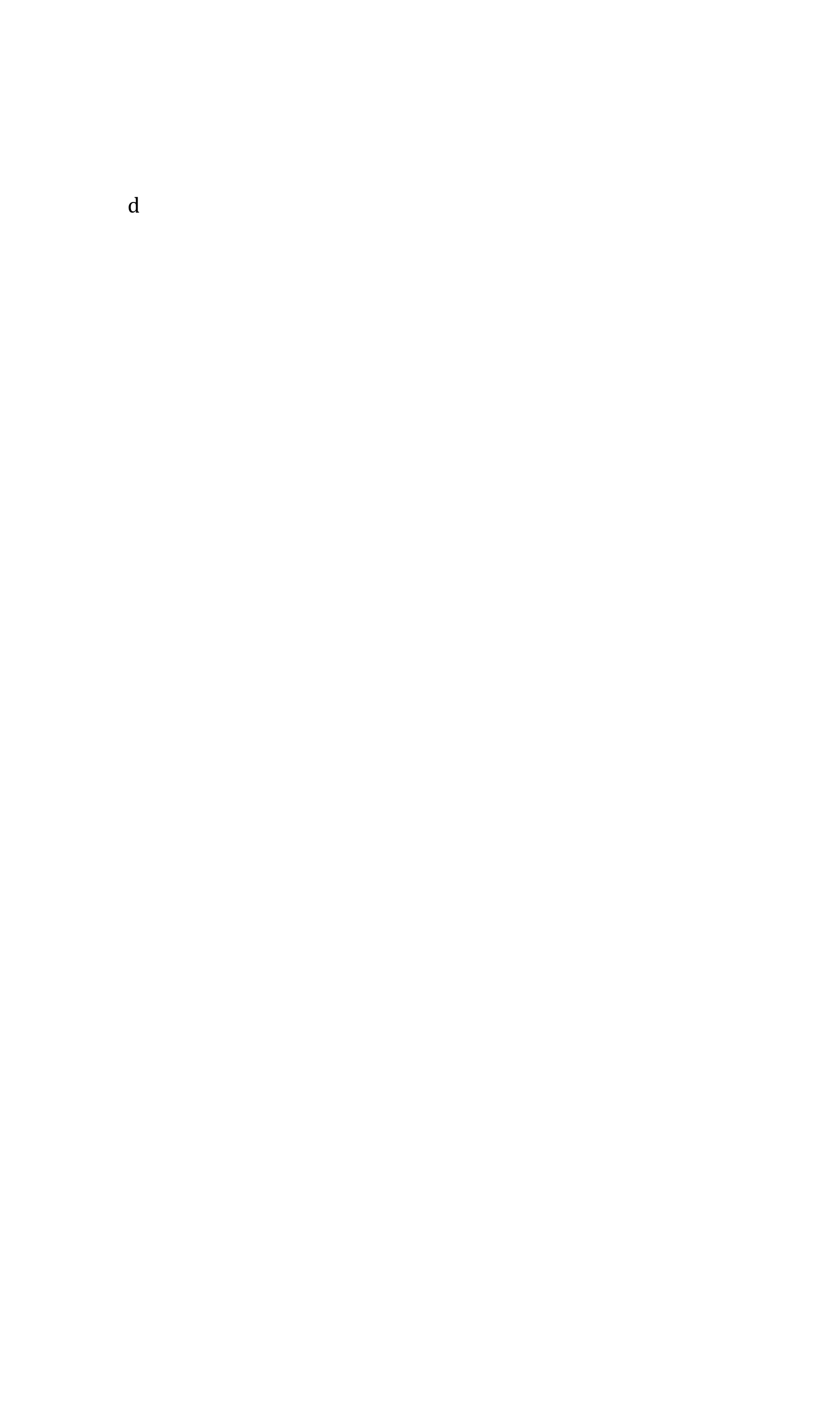
b

f

d

g

1. Eliminating

f

d

g

1. Eliminating

d

g

1. Eliminating

g

1. Finally the is:

c

b

a

g

d

g

e

Maximal Cliques are:

Therefore size of largest maximal clique is .

Hence under elimination order of the graph is

.

I have tried out other elimination orders, but elimination order gave me the minimal treewidth.

Hence of graph

Edges required to triangulate the graph are: .